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Roller chain drive analysis: simplified modeling and analysis of the dynamic effects of meshing

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Summary. Transverse vibration of a roller chain and the effects of the interaction with sprockets is examined analytically. Modeling the chain as a uniform string, we consider the meshing process either as a moving boundary, or as boundary impacts and apply perturbation methods to predict dynamical responses.

Introduction

We analyze the transverse vibrations of a traveling string subjected to the excitation at the boundaries that is typical of roller chain drives. Meshing of a roller chain with a sprocket causes noise and vibration, and by modeling the chain as a moving, heavy, uniform string we present two approaches to how the dynamical effects of meshing can be analyzed. Modeling a roller chain as a uniform heavy string and studying the transverse vibrations for simple harmonic excitation was considered already by [1]. Traveling strings in general is a rich field which has been subject to many studies, see e.g. the comprehensive review by [2]. In a roller chain drive, the chain is subjected to complicated excitation when it interacts with the sprockets, because of the discrete nature of the chain and sprockets, known as polygonal action. A detailed study of the kinematic characteristics of roller chain drives is given in [3] and studies of impact intensities between roller and sprockets were carried out by [4]. Linear transverse vibrations of moving strings was considered by [5] and also weak nonlinearity has been investigated [6]. Here, it is demonstrated how the interaction between a chain and two sprockets is nonlinear. We present a simplified model of the chain, and two methods for how the system can be analyzed by considering the meshing process as equivalent to a) kinematically forced boundary conditions, or b) boundary impacts.

Physical system

Figure 1a shows a schematic of a chain drive consisting of two sprockets and illustrates the discrete nature of the drive. The driving sprocket rotates at an angular velocity $\dot{\theta}_1$ and the chain travels axially with velocity v. Relative velocities between the approaching roller, $\oplus$ and the sprocket seat $\times$ causes impacts, and this meshing process is a significant source of noise and vibration. This and e.g. a periodically varying chain tension are inherent for roller chain drives, owing to the sprockets forming polygons instead of circles. Meshing is shown in greater detail in Figure 1b, and here it is seen that the chain and sprocket interaction is nonlinear, because it depends on the vibration of the axially moving chain. Similarly, guide bars installed to limit transverse vibrations of chain spans may also introduce impact effects. We aim at analyzing the transverse vibration of the chain when subjected to this characteristic excitation, specifically the *effects* of meshing, and we consider only the dynamics of a single chain span, i.e. the tight chain span is analyzed while the dynamic responses of the sprockets and other chain drive components are not considered.

Analysis

Fundamental mathematical model

Our first approximations for a model to analyze transverse vibrations of a roller chain span is to assume that the chain can be modeled as a uniform heavy string, with mass per unit length m, i.e. the periodic variation of chain density and cross sectional area are both neglected, and the bending stiffness is assumed to be vanishing. Chain tension T and length L are considered to be constant, and we assume $\dot{\theta}_1$ is given such that the axial velocity v is constant and lower than the speed

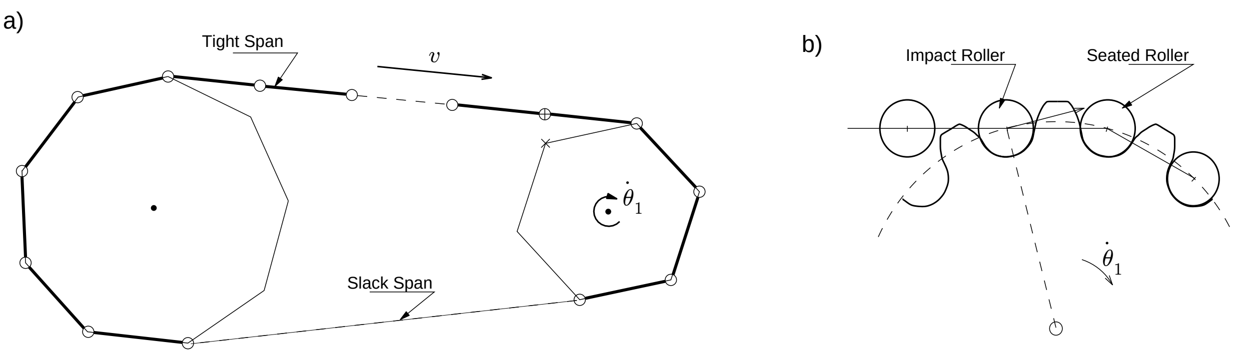

Figure 1: a) Principal sketch of a chain drive consisting of two sprockets. Sprockets form polygons, and impact happens between the approaching chain roller $\oplus$ and the sprocket seat $\times$. b) Sketch showing a more realistic drawing of the situation where the impact roller, suspended between the seated roller and the remaining chain span gets into contact with the sprocket seat.

of propagating transverse waves, $v_0 = \sqrt{T/m}$. The equation of motion governing the transverse displacement $w(x,t)$ of the traveling string is [1]:

$$w_{,tt} + 2vw_{,xt} + (v_0^2 - v^2)w_{,xx} + N(w) = 0, \tag{1}$$

where a comma subscript denotes partial differentiation and $N(w)$ has been added to contain nonlinear operators of w. With $N = 0$ equation (1) is linear, but nonlinearity must be included if e.g. finite vibration amplitudes between axially fixed boundaries are to be considered, as could be necessary for obtaining realistic solutions at resonant excitation.

Kinematically forced boundaries

First, we analyze the effect of chain and sprocket meshing by making the string kinematically forced, that is by specifying boundary motion as a function of time, i.e. $w(0,t) = \delta_1(t)$ and $w(L,t) = \delta_2(t)$, as illustrated in Figure 2a. Such a system is conveniently analyzed by applying the transformation $w(x,t) = \tilde{w}(x,t) + \delta_1(t)(1 - x/L) + \delta_2(t)x/L$, which renders the boundary conditions homogeneous while the excitation at the boundaries is moved to the differential equation for $\tilde{w}$. When $N(w) = 0$ equation (1) has exact closed form solutions for arbitrary excitation and this will be used as a basis for a perturbation solution when nonlinearity is included.

Boundary impact forces

As an alternative to the kinematic forcing described above, we analyze the effect of meshing by specifying the position of an impact surface $w_0 = w_0(x,t)$, allowing for contact between the chain and sprocket away from the boundaries. This contact will depend on the vibration of the chain and is a nonlinear force $g(w - w_0)$ of the discontinous clearance type. Boundary conditions for equation (1) are homogeneous in this case, and of special interest are the effects of nonlinear terms of the clearance type, see Figure 2b. A system of this type can be analyzed through, e.g., discontinuous transformations, by doing a Fourier expansion of the nonlinear terms, or by use of numerical methods such as continuation. As a first step, we aim towards a perturbation solution of (1) when nonlinearity of the clearance type is included, and treated analytically by Fourier expansion.

Expected results

By making simplified mathematical models of a chain span and examining specific phenomena, we aim at contributing with simple predictions that are relevant for understanding the complexity of full chain drive systems. With perturbation solutions obtained by the approaches outlined above, we aim towards analytical predictions of frequency responses, and effects of nonlinearity to be investigated for, e.g., slow- medium- and fast excitation frequencies. Approximate analytical results will be tested against numerical simulation of the simple models. Furthermore we aim to examine the validity of the physical approximations by comparing analytical results with detailed Multibody Dynamics Simulations, and possibly with existing experimental results.

Conclusions

This work is in progress. We expect to find an analytical expression for the transverse vibrations of a traveling string subjected to the characteristic excitation from chain and sprocket meshing.

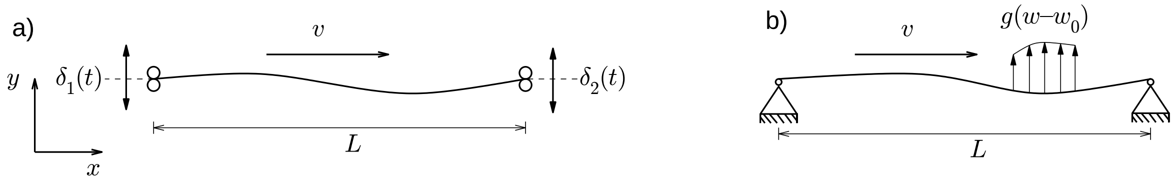

Figure 2: The chain is modeled as a moving string and the effect of meshing is examined by a) specifying the motion of the boundaries, or b) by specifying the motion of a contact surface $w_0(x,t)$ away from the boundaries with a nonlinear forcing of the clearance type $g(w - w_0)$ used to examine the effects of meshing.

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